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SIMPLE TIME-PHASE/FREQUENCY LOCKED LOOP, FUNCTIONING AS LOW PASS TIME-DIGITAL FILTER

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Abstract

This article describes development, time and frequency analysis, application and simulation of a recursive Time-Phase/Frequency Locked Loop (TP/FLL). The described TP/FLL is based on the measurement and processing of the time-differences between periods of the input and output signals. It may function either as a Phase Locked Loop or a Frequency Locked Loop. TP/FLL is very simple for implementation because it is defined only by one system parameter and one outside control word. Mathematical description, analysis of stability conditions and properties of TP/TFLL are performed using Z transform. Using mathematical analyses and simulations, it is shown that TP/FLL is widely applicable for the phase shifting of pulse signal. Although very simple, TP/FLL naturally possesses the property of low pass digital filter. Special emphasis is given to the analysis and development of a Time-low pass digital filter intended for the filtering of pulse signal periods, using the Mat-lab tools intended for development of classical digital filter.

Keywords: Time-digital filters, Digital circuits, Frequency locked loops, Phase locked loops, Linear discrete system.

INTRODUCTION

Articles [1-7] are based on the processing of input and output signal periods and the differences between them. These articles illustrate the great possibilities of this approach in a very wide range of applications in the field of electronics, telecommunications, measurement, control and other fields where electronics are used. In articles [1-3] the theoretical foundations and practical examples are presented in the development of a new type of digital filters, which we called Time-digital filters. Unlike classic digital filters that process signal amplitude, these filters process only time, i.e. the periods and time differences between the input and output periods. Time Phase Locked Loops (TPLL) and Time Frequency Locked Loops (TFLL) are described in articles [4-7]. These new types of PLLs and FLLs are similar to digital filters in terms of their mathematical form and method of analysis, and compared to classic PLLs and FLLs, they offer a wider range of applications with new features. The articles shown in the references represent both the theoretical and practical basis for the further development and application of this approach in many scientific directions.

This article describes a very simple Time- Phase/Frequency Locked Loop model, which can function either as a phase or frequency locked loop. The model indicates an important feature of these systems i.e. that processing of time differences only, without the period of the input and output signal, can enable digital filtering of the input signal period. Note that in articles [1-3] the Time FLL parameters were replaced with the coefficients of the pre-designed digital filter and thus ensured that the Time FLL functions as a digital filter of periods. Therefore, in the development of the Time-digital filter, we relied on the theory of the classical digital filters. In this work, the theory of the classical digital filters is not used. But it is worth to emphasize that instead of periods, TP/FLL performs

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processing of time differences. This fact ensures that the described model functions as a low pass digital filter. The article also shows that with external control we can incorporate additional system functions into TP/FLL or improve the existing ones. Although the described system is very simple, it is shown in the article that TP/FLL has powerful performance for phase shifting of pulse signals and time-digital filtering of the pulse signal periods.

The articles and books [8-17] are used as the theoretical and mathematical base.

DESCRIPTION AND ANALYSIS OF TP/FLL

The general case of the input and output signals Sin and Sop for TP/FLL is shown in Fig. 1. Periods TI_k and TO_k, as well as the time differences τ_k and time intervals T_k, occur at discrete times t_k, t_{k+1} and t_{k+2}, which are defined by the falling edges of the pulses of Sop. The difference equation of TP/FLL is presented in eq. (1), where "m" is the system

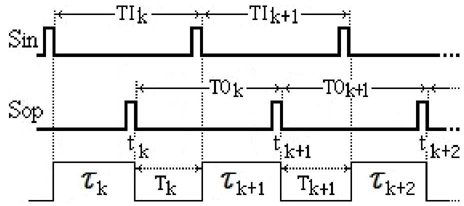

Fig. 1. *Time relations between the input and output variables of TP/FLL.*

parameter of TP/FLL and "T_c" is the time constant which is outside control word. It will be entered into the system automatically from the outside. One additional natural relation between the time variables, which yields from Fig. 1, is shown in eq. (2). Note that the difference eq. (1) contains only one system parameter "m" and that it contains neither the input nor the output period, but only the time difference τ_k, which is presented in Fig. (1). However, we see that the variable τ_k defined by eq. (2) contains both the output and input periods, so that eq. (1) represents a recursive form, describing the linear discrete system TP/FLL. The fact that eq. (1) is very simple enables its simple implementation. In order to make analysis of TP/FLL let us find the Z transforms of eqs. (1) and (2). They are presented in eqs. (3) and (4). Note that TO_0, and τ_0 in eqs. (3) and (4) are the initial conditions of TO_k and τ_k. If we calculate $\tau(z)$ from eq. (4) and replace it in eq. (3), we can calculate TO(z), which is shown in eq. (5). If we substitute now TO(z) from eq. (5) into eq. (4), we can found out the expression for $\tau(z)$, shown in eq. (6). Two transfer functions describing TP/FLL, which are given by eqs. (7) and (8), can be defined from eqs. (5) and (6).

$$TO_{k+1} = m \cdot \tau_k + T_c \tag{1}$$

$$\tau_{k+1} = \tau_k + TO_k - TI_k \tag{2}$$

$$zTO(z) - zTO_0 = m \cdot \tau(z) + zT_c / (z-1) \tag{3}$$

$$z \cdot \tau(z) - z \cdot \tau_0 = \tau(z) + TO(z) - TI(z) \tag{4}$$

$$TO(z) = TI(z)\frac{-m}{z^2 - z - m} + \frac{zT_c}{z^2 - z - m} + \frac{zm\tau_0}{z^2 - z - m} \tag{5}$$

$$\tau(z) = TI(z)\frac{-z}{z^2 - z - m} + \frac{zT_c}{(z-1)(z^2 - z - m)} + \frac{z^2\tau_0}{z-1} \tag{6}$$

$$H_{TO}(z) = \frac{TO(z)}{TI(z)} = \frac{-m}{z^2 - z - m} \tag{7}$$

$$H_{\tau}(z) = \frac{\tau(z)}{TI(z)} = \frac{-z}{z^2 - z - m} \tag{8}$$

In order to analyze TP/FLL, let us suppose that the step function TI(k)=TI=constant is applied to the input. Z transform of TI(k) is $TI(z)=TI \cdot z/(z-1)$. If we enter TI(z) into eq. (5), using the final value theorem, it is possible to find $TO_\infty = \lim TO(k)$ if $k \to \infty$, using TO(z). This is shown in eq. (9). It comes out from eq. (9) that TP/FLL can function as a TFLL unconditionally.

$$TO_{\infty} = \lim[(z-1)\cdot TO(z)]_{z\to 1} = TI \qquad (9)$$

Let us now check if the system can operate as a TPLL. In order to do that, let us find out the final value of *τ(k)* if $k\to\infty$, i.e. in case when the system is in the stable state. The system can possess the properties of a TPLL if the final value of *τ(k)* is equal to zero. Providing that the step function TI(k)=TI is applied to the input, Tl(z) in eq. (6) should be substituted by TI·z/(z-1). We can find out the final value $\tau_{\infty}=\lim[\tau(k)]_{k\to\infty}$, using the final value theorem in Z transform notation $\tau_{\infty}=\lim[(z-1)\cdot\tau(z)]_{z\to 1}$. Using this expression, we can get τ_{∞}, shown in eq. (10). As we can see from eq. (10), time difference τ_{∞} is not equal to zero in the stable state of TP/FLL, although it does not depend on the initial conditions τ_0 and TO_0. But τ_{∞} can be equal to zero providing that control word T_c is equal to the constant input period TI. That means TP/FLL can possess the properties of a TPLL. It is of interest to note that the TP/FLL can function as a TPLL even in the case where the period of the input signal varies in time. In order to achieve this, it is necessary to continuously measure the input period TI and use it as the control word T_c.

$$\tau_{\infty} = \frac{TI - T_c}{m} \qquad (10)$$

All the previous conclusions, including the results given by eqs. (9) and (10), are valid only if the system is stable. TP/FLL is the stable system if the poles $|z_1| < 1$ and $|z_2| < 1$, where z_1 and z_2 are the zeros of the polynomial z^2-z-m in eq. (7) or in eq. (8). The zeros z_1 and z_2 are shown in eq. (11).

$$z_{1/2} = 0.5 \pm \sqrt{0.25 + m} \qquad (11)$$

The conditions $|z_1| < 1$ and $|z_2| < 1$ define the region in the plane of parameter “m”, where TFLL is the stable system. This region of parameter “m” is presented in eq. (12).

$$-1 \prec m \prec 0 \qquad (12)$$

ANALYSIS OF TP/FLL IN PULSE TRAIN PHASE SHIFTING APPLICATIONS

Let us remember that τ_{∞} in eq. (10) represents the delay time of the output signal pulses with respect to the input signal pulses, when the TP/FLL is in stable state. The delay time τ_{∞} can easily be converted into the phase delay Ph according to the expressions $Ph=360°\cdot(\tau_{\infty}/TO_{\infty})$ [°] or $Ph = 2\pi\cdot(\tau_{\infty}/TO_{\infty})$ [rad]. It can be seen from eq. (10) that there are two parameters "m" and "T_c" through which we can influence the time or phase delay. The first is the system parameter "m" and the second is the external parameter "T_c" which can be changed as needed in order to control the delay.

In order to investigate the delay performances of TP/FLL, let us now make the simulation of TO_k and τ_k for the different values of “m” and “T_c”. All time variable will be expressed in time units (t.u.). Note that t.u. can be μs, ms, or any other time unit, assuming the same time unit for all time variables. For simplicity, "t.u." units are omitted from the diagram. The simulations of TO_k, and τ_k , for TI_k=10 [t.u.] and T_c=20 [t.u.] are shown in Fig. 2. The simulations are made for three combinations of the parameter “m”, and for the initial conditions, which are presented in Fig. 2. We can see that $TO_{1\infty}$, $TO_{2\infty}$ and $TO_{3\infty}$ are equal to TI in accordance with eq. (9). Due to fact that all of “m” in Fig. (2) are different and satisfy eq. (12), the time delays $\tau_{1\infty}$, $\tau_{2\infty}$ and $\tau_{1\infty}$, tend to different finite values. Let's check if the values of time delays obtained by simulations are correct. According

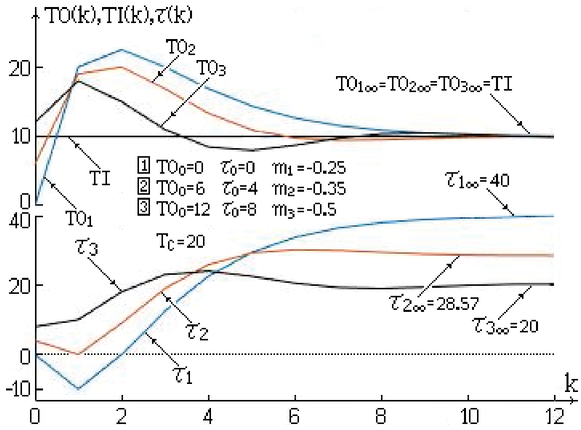

***Fig. 2.** The delay characteristic of TP/FLL depending on the system parameter "m" for T_c=constant=20 t.u.*

to eq. (10) $\tau_{1\infty}=(TI-Tc)/m_1=(10-20)/(-0.25)=40$ [t.u.], $\tau_{2\infty} = (TI-Tc)/m_2 = (10-20)/(-0.35)=28.57$ [t.u.] and $\tau_{3\infty} = (TI-Tc)/m_3 = (10-20)/(-0.5) = 20$ [t.u.]. All calculated delays agree with those obtained by simulation in Fig. 2, which proves the correctness of all previous mathematical analyzes and simulations.

Using simulations, let us now illustrate the influence of the external parameter T_c on the delay of the pulse train for the case when the system parameter m=constant=-0.25. The simulations are made for three combinations of the control word T_c, and for the initial conditions, which are presented in Fig. 3. We can see that $TO_{1\infty}$, $TO_{2\infty}$ and $TO_{3\infty}$ are equal to TI in accordance with eq. (9). Due to fact that all of T_c used in $TO_{1\infty}$, $TO_{2\infty}$ and $TO_{3\infty}$ in Fig. (3) are different, the time delays $\tau_{1\infty}$, $\tau_{2\infty}$ and $\tau_{3\infty}$, tend to different finite values. Let's check if the values of time delays obtained by simulations are correct. According to eq. (10) $\tau_{1\infty}=(TI-T_{c1})/m=(10-15)/(-0.25)=20$ [t.u.], $\tau_{2\infty} = (TI-T_{c2})/m = (10-20)/(-0.25)=40$ [t.u.] and $\tau_{3\infty} = (TI-TI_{c3})/m = (10-25)/(-0.25) = 60$ [t.u.]. All calculated delays agree with those obtained by simulation in Fig. 3, which proves the correctness of all previous mathematical analyzes and simulations.

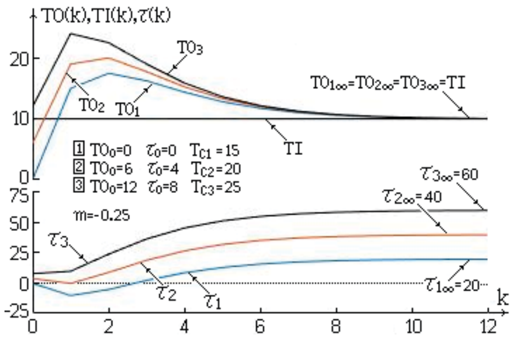

***Fig. 3.** The delay characteristic of TP/FLL depending on the outside parameter T_c for m=constant=-0.25 t.u.*

According to eq. (10), if control word T_c=TI, time delay $\tau_\infty=0$, no matter of the value of the system parameter "m". The simulation of TO_k and τ_k for three different value of "m" and for T_c=TI=10 [t.u.] is presented in Fig. 4. According to eq. (10), we can conclude that if T_c>TI, $\tau_\infty>0$. Based on the definition of time difference in Fig. 1, if $\tau_\infty>0$, it means that the pulses of the output signal lag behind the pulses of the input signal. If $T_c<0$, then $\tau_\infty<0$ which means that the output pulses precede the input signal pulses. Finally, if T_c=TI, as it is the case in Fig. 4, the phase difference between the output and input pulses is equal to zero. In the TP/FLL implementation, we can always measure the period of the input signal and use it instead of the control word T_c. In that case, even if the input period is variable in time, we can ensure that the phase difference between the output and input pulses tends to zero. Therefore, although the described model is very simple to implement, it possesses the properties of a Phase Locked Loop, even in the case of a dynamic change in the period of the input signal.

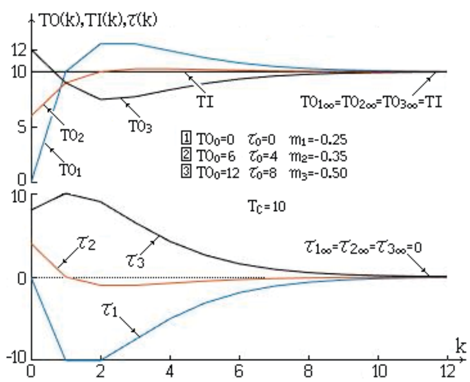

***Fig. 4.** TP/FLL functions as Time-Phase Locked Loop for T_c=TI=10 [t.u.], no matter of the value of parameter "m".*

TP/FLL PERFORMANCES IN DIGITAL FILTERING OF PERIODS

As it is stated in the Introduction, Time-digital filters in refs. [1-3] were developed by changing the system parameters with the coefficients of a predefined classical digital filter. However, the simple model TP/FLL possesses naturally the low pass time-digital filter characteristics. The question arises whether it is possible to change the filter characteristics of TP/FLL, as it is possible in the case of time-digital filters, described in refs. [1-3]? To get the answer, let us define vectors "b" and "a" using the transfer function of TP/FLL, shown in eq. (7). Using the rules of the application Matlab software, intended to digital filters, vectors "b" and "a" are defined and shown in eq. (13). Using command "freqz (b, a, 1024, fs)", where the sample frequency fs=10000 Hz, the frequency responses of TP/FLL is determined and presented in Fig. 5.

$$b = [0 \quad 0 \quad -m], \qquad a = [1 \quad -1 \quad -m] \qquad (13)$$

The magnitudes are determined for five values
of "m" and the phase is presented for m=-0.05. The frequency response is presented for the half of the sample rate. It can be seen in Fig. 5 that the filtering characteristic of TP/FLL can be changed by the system parameter "m". Linear magnitudes of the TP/FLL frequency response, for the same values of parameter "m" as in Fig. 5, are presented in Fig. 6. Using Fig. 6, the bandwidths of the filter, for different parameter "m", can be easily determined. In order to determine the bandwidth of TP/FLL, in Fig. 6 is marked value 0.708 which corresponds approximately to -3 dB. For example, if m=-0.35, the filter bandwidth is about 1320 Hz in Fig. 6. This bandwidth will be used in the following analysis. According to Fig. 6, increasing of "m" increases the bandwidth of the filter, but decreases the slope of magnitudes, which decreases the attenuation within the stop bandwidth of the filter. With classic digital filters and time-digital filters described in refs. [1-3], the slopes of the magnitudes increase with increasing of the system order.

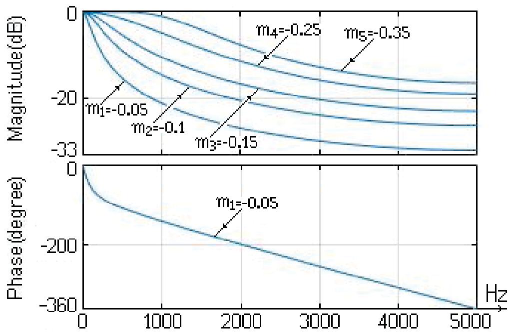

Fig. 5. *Magnitudes and Phase of the TP/FLL frequency response for the different values of parameter "m".*

Let us demonstrate the filter characteristics of TP/FLL for m=-0.35. Although the slope of the magnitude in Fig. 6 is the smallest for m=-0.35, this case was purposely chosen because it will provide very clear insight into the time and frequency domain of the TP/FL operation. Suppose that the input period TI_{k+1} is defined as TI(k+1) = 10 + S_1(k + S_2(k) [t.u.], where $S_1(k) = 7\cdot\sin[2\pi/fs\cdot f_1\cdot k]$ [t.u.] and $S_2(k) = 7\cdot\sin[2\pi/f_s\cdot f_2\cdot k]$ [t.u.]. Suppose that the values of the frequencies are f_1=500 Hz and f_2=4500 Hz. Note that the frequency f_1 is inside of the bandwidth of TP/FLL, which is 1320 Hz, shown in Fig. 6. But the frequency f_2 is far away from the bandwidth of TP/FLL. Since f_s=10000 Hz, the first step in this presentation

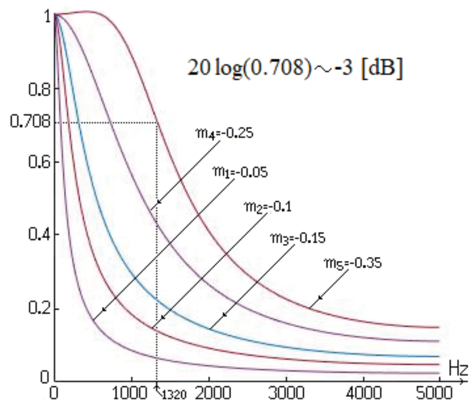

Fig. 6. *Linear Magnitudes of the TP/FLL frequency response for the same values of parameter "m" as in Fig. 5.*

is to form vector TI of 10000 values of TI, using the above equation for TI_{k+1}. Based on the vector TI, the output period vector TO = filter (b, a, TI) is determined. After that, using the "fft" command, the input and output vectors of TP/FLL are formed as X = fft (TI) and Y = fft (TO). Finally, using the command "stem", stem (abs (X)) and stem (abs (Y)), the spectrums of the input and output periods are presented in Fig. 7. These spectrums present the absolute values of the amplitudes, covering the whole sample rate. They appear as positive values in the symmetric second half of the sample rate. It is visible in Fig. 7 that signal S_1 at 500 Hz is not attenuated, since f_1 is inside of

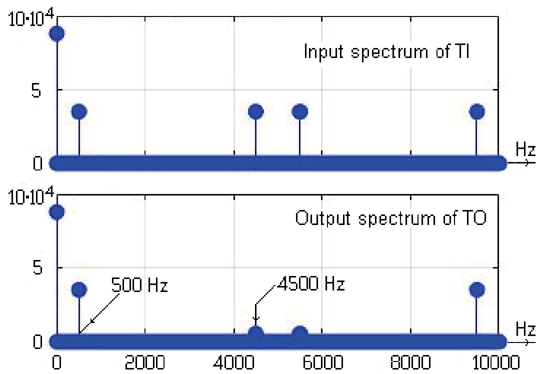

Fig. 7. *The input and output spectrums of TI and TO.*

the bandwidth. This is in agreement with magnitude of the TP/FLL for m=-0.35, shown in Fig. 6, since at f_1=500 Hz, the attenuation is close to zero. At the same time signal S_2 at 4500 Hz is almost completely suppressed.

CONCLUSION

The described design and analysis of the TP/FLL represents a further elaboration on the recently described theory, design and implementation, based on the processing of input and output periods and the time differences between them, refs. [1 to 7]. In this article, it is shown that even a very simple algorithm, based only on time differences, can function either as a low pass time-digital filter or as a powerful pulse train phase shifter.

For this original approach, only one system parameter and one external control word were used. The system parameter affects the operation of the TP/FLL in both the time and frequency domains, while the control word is a powerful tool for shaping the output signal in the time domain, but without any effect on the frequency domain of the TP/FLL operation, that is, on its filter characteristics. Given this feature, the application of these mechanisms to other applications could not be researched in this article due to the limited space, which will certainly be the subject of future research.

It is especially important to point out one new specificity of this approach, in which the coefficients of classical digital filters were not used for the development of the described digital filter, as in the development of time-digital filters, described in refs. [1 to 3]. This fact also imposes the need for further research in the direction of developing new time-digital filters based on this principle, which would have a wider range of applications.

Acknowledgements

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REFERENCE

[1] Dj. M. Perišić, "New kind of IIR digital filters intended for pulse period filtering", Rev. Roum. Sci. Techn. – Électrotechn. et Énerg., Vol. 69, 1, pp. 61–66, 2024.

[2] Dj. M. Perišić, "Digital filters intended for pulse signal periods", Rev. Roum. Sci. Techn. – Électrotechn. et Énerg., Vol. 67, *2*, pp. 161–166, 2022.

[3] Dj. M. Perišić, "Frequency locked loops of the third and higher order", Rev. Roum. Sci. Techn. – Électrotechn. et Énerg., Vol. 66, No. 4 pp. 261–266, (2021).

[4] Dj. M. Perišić, M. Bojović, "Multipurpose Time Recursive PLL ", Rev. Roum. Sci. Techn. – Électrotechn. et Énerg., Vol.**61**, 3, pp. 283–288, 2016.

[5] Dj. M. Perišić, A. Žorić, M. Perišić, D. Mitić, "Analysis and Application of FLL

based on the Processing of the Input and Output Periods", Automatika 57 (2016).
[6] Dj. M. Perišić, M. Perišić, D. Mitić, M. Vasić "Time Recursive Frequency Locked Loop for the tracking applications", Rev. Roum. Sci. Techn. – Électrotechn. et Énerg., **60**, 2, pp. 195–203, 2015.
[7] Dj. M. Perisic, A. Zoric, M. Perisic, V. Arsenovic, Lj. Lazic, "Recursive PLL based on the Measurement and Processing of Time", Electronics and Electrical Engineering, Vol. 20, No. 5, pp. 33-36, (2014).
[8]. D. Jovcic, "Phase locked loop system for FACTS", IEEE Transaction on Power System, **18**, pp. 2185-2192 (2003).
[9] G. Bianchi, "Phase-Locked Loop Synthesizer Simulation", *Nc-Hill, Inc.*, New York, USA (2005).
[10] M, Gardner, "Phase lock techniques", *Hoboken*, Wiley-Interscience, (2005).
[11] B. D. Talbot, "Frequency Acquisition Techniques for Phase Locked Loops", Wiley-IEEE Press, pp. 224 (2012).
[12] R. Vich, "Z Transform Theory and Application (Mathematics and Applications)", *Ed. Springer*, (1987-first edition).
[13] A. K. Maini, "Digital Electronics, principles, devices and applications", John Wley and Sons, Ltd, 2007.
[14] S. Winder, "Analog and Digital Filter Design" (second edition), Copyright©2002 Elsevier Inc., 2002).
[15]. S. W. Smith, "Digital Signal Processing" (second edition), California Technical Publishing, (1999).
[16]. W. F. Egan, "Phase-Lock Basics" (second edition), John Wiley and Sons, (2008).
[17]. C. B. Fledderman, "Introduction to Electrical and Computer Engineering", Prentis Hall, (2002).